සීමාව ගැටළු විසඳීමේ පහත ප්‍රතිඵල භාවිතා කළ හැක.

සීමාවේ මූලික නීති :-

01. තනිකර නියතයක සීමාව, එම නියතයම වේ.

$$\lim_{x \to a} C = C$$

උදා :- (1) $\lim_{x \to 0} 5 = 5$ (2) $\lim_{x \to \infty} 7 = 7$

02. නියතයක්, ශ්‍රිතයකින් ගුණ වී ඇතිවිට, එම නියතය පිටතට ගත හැක.

$$\lim_{x \to a} C.f(x) = C \lim_{x \to a} f(x)$$

උදා :- $\lim_{x \to 1} 5x^2 = 5 \lim_{x \to 1} x^2 = 5.1^2 = 5$ //

03. එකතු කිරීමත්, අඩු කිරීමත්, වැඩි කිරීමත්, බෙදීමත් ඇති විට, ඒවා වෙන්කර වෙන, වෙනම සීමාව යෙදිය හැක.

$$\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

$$\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$$

$$\lim_{x \to a} f(x).g(x) = \lim_{x \to a} f(x) . \lim_{x \to a} g(x)$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$$

උදා :- (1) $\lim_{x \to 0} \frac{\sin 5x}{\sin 3x}$

$$\lim_{x \to 0} \frac{\frac{\sin 5x}{x}}{\frac{\sin 3x}{x}}$$

$$\frac{\lim_{x \to 0} \frac{\sin 5x}{5x} . 5}{\lim_{x \to 0} \frac{\sin 3x}{3x} . 3} = \frac{1.5}{1.3} = \frac{5}{3}$$ //

04. **බලයක් ඇති විට**

$$\lim_{x \to a} [f(x)]^n = \left(\lim_{x \to a} f(x) \right)^n$$

05.

$$\lim_{x \to a} \frac{1}{f(x)} = \frac{1}{\lim_{x \to a} f(x)}$$

06.

$$\lim_{x \to a} |f(x)| = \left| \lim_{x \to a} f(x) \right|$$

සිද්ධාන්තමය ගැටළු

01. පහත සීමා සොයන්න.

(1) $\lim_{x \to 1} \frac{x^2 - 4x + 3}{x^2 - 1}$ (2) $\lim_{x \to 2} \frac{x^2 - 6x + 8}{x^2 - 5x + 6}$ (3) $\lim_{x \to -1} \frac{x^3 + 1}{x^4 - 1}$

(4) $\lim_{x \to -1} \frac{x^3 + 5x^2 + 7x + 3}{x + 1}$ (5) $\lim_{x \to -2} \frac{x^3 + 2x^2 + 7x + 14}{x^2 - 4}$ (6) $\lim_{x \to 1} \frac{x^3 + 5x^2 + 3x - 9}{x^3 - 1}$

02. (1) $\lim_{x \to 1} \frac{x^7 - 1}{x - 1}$ (2) $\lim_{x \to 2} \frac{x^5 - 32}{x - 2}$ (3) $\lim_{x \to 3} \frac{x^4 - 81}{x - 3}$

(4) $\lim_{x \to 64} \frac{x^{1/2} - 8}{x - 64}$ (5) $\lim_{x \to 3} \frac{x - 3}{x^3 - 27}$ (6) $\lim_{x \to 5} \frac{x - 5}{x^3 - 125}$

(7) $\lim_{x \to 1} \frac{x^9 - 1}{x^5 - 1}$ (8) $\lim_{x \to 2} \frac{x^7 - 128}{x^5 - 32}$ (9) $\lim_{x \to 1} \frac{x^{1/5} - 1}{\sqrt{x} - 1}$

(10) $\lim_{x \to 1} \frac{x^7 - x^{-7}}{x - 1}$ (11) $\lim_{x \to 1} \frac{x^{11} - 1}{1 - x^7}$ (12) $\lim_{x \to 1} \frac{x^m - 1}{x^n - 1}$

03. (1) $\lim_{x \to 2} \frac{\sqrt{x + 5} - \sqrt{7}}{x - 2}$ (2) $\lim_{x \to 0} \frac{x}{\sqrt{x + 3} - \sqrt{3}}$

(3) $\lim_{x \to 5} \frac{\sqrt{9 - x} - 2}{\sqrt{x} - \sqrt{5}}$ (4) $\lim_{x \to -1} \frac{\sqrt{8 - x} - 3}{\sqrt{2 + x} - 1}$

04. (1) $\lim_{x \to \pi/6} \frac{2 \sin^2 x - 3 \sin x + 1}{2 \sin x - 1}$ (2) $\lim_{x \to \tan^{-1} 2} \frac{\tan^2 x - 3 \tan x + 2}{\tan x - 2}$

(3) $\lim_{x \to \pi/3} \frac{4 \cos^2 x - 4 \cos x + 1}{2 \cos x - 1}$

05. (1) $\lim\limits_{x \to 0} \dfrac{\sin 7x}{x}$

(2) $\lim\limits_{x \to 0} \dfrac{\sin 9x + \sin x}{x}$

(3) $\lim\limits_{x \to 0} \dfrac{\sin^3 x}{x^3}$

(4) $\lim\limits_{x \to 0} \dfrac{\sin^2 5x}{x^2}$

(5) $\lim\limits_{x \to 0} \dfrac{\sin^4 2x}{x^4}$

(6) $\lim\limits_{x \to 0} \dfrac{\sin 7x - \sin 2x}{\sin 3x + 4x}$

(7) $\lim\limits_{x \to 0} \dfrac{1 - \cos 4x}{x^2}$

(8) $\lim\limits_{x \to 0} \dfrac{1 - \cos 8x}{1 - \cos 2x}$

(9) $\lim\limits_{x \to 0} \dfrac{\sqrt{2 + \sin x} - \sqrt{2 - \sin x}}{x}$

(10) $\lim\limits_{x \to 0} \dfrac{\sqrt{7 + \sin^2 x} - \sqrt{7}}{x^2}$

(11) $\lim\limits_{x \to 0} \dfrac{\sin 7x + \sin 4x - \sin 5x}{4x - \sin 3x}$

(12) $\lim\limits_{x \to 0} \dfrac{1 - \cos 2x + \sin^2 3x}{\sin^2 x}$

(13) $\lim\limits_{x \to 0} \dfrac{\tan x}{x}$

(14) $\lim\limits_{x \to 0} \dfrac{\tan^2 2x}{x^2}$

(15) $\lim\limits_{x \to 0} \dfrac{x^2}{\sqrt{2 + \tan^2 x} - \sqrt{2}}$

(16) $\lim\limits_{x \to 0} \dfrac{1 - x \sin 3x - \cos 4x}{x^2}$

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06. (1) $\lim\limits_{x \to 0} \dfrac{\sin^{-1} x}{x}$

(2) $\lim\limits_{x \to 0} \dfrac{x}{\tan^{-1} x}$

(3) $\lim\limits_{x \to 0} \dfrac{\sin 3x}{\sin^{-1} x}$

(2) $\lim\limits_{x \to 3} \dfrac{\tan^{-1} x\,(x-3)}{(x-3)}$

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07. (1) $\lim\limits_{x \to 0} \dfrac{(1+4x)^5 - 1}{x}$

(2) $\lim\limits_{x \to 0} \dfrac{(1+2x)^{\frac{1}{4}} - 1}{x}$

(3) $\lim\limits_{x \to 0} \dfrac{(x+2)^7 - 128}{(x+2)^4 - 16}$

(4) $\lim\limits_{x \to 0} \dfrac{(1+x)^9 - 1}{\sqrt{(x+2)} - \sqrt{2}}$

(5) $\lim\limits_{x \to 0} \dfrac{(3+x)^5 - 243}{\sin 2x}$

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08. (1) $\lim\limits_{x \to \infty} x^3 - 4x^2 - x + 1$

(2) $\lim\limits_{x \to \infty} 1 - 2x - x^5$

(3) $\lim\limits_{x \to \infty} \dfrac{3x^2 + 7x + 1}{x^2 + 2x + 5}$

(4) $\lim\limits_{x \to \infty} \dfrac{x^3 + 4x^2 + 3x + 1}{2x^2 + 7x + 1}$

09. (1) $\lim\limits_{x \to 0} \dfrac{e^x - 1}{x}$ (2) $\lim\limits_{x \to 0} \dfrac{e^{4x} - 1}{x}$ (3) $\lim\limits_{x \to 0} \dfrac{e^{px} - e^{Qx}}{x}$

(4) $\lim\limits_{x \to 0} \dfrac{e^{2x} - 1}{e^{5x} - 1}$ (5) $\lim\limits_{x \to 0} \dfrac{e^{\sin x} - 1}{x}$

10. (1) $\lim\limits_{x \to 0} \dfrac{2^x - 1}{x}$ (2) $\lim\limits_{x \to 0} \dfrac{5^x - 1}{x}$

(3) $\lim\limits_{x \to 0} \dfrac{4^x - 3^x}{x}$ (4) $\lim\limits_{x \to 0} \dfrac{a^x - 1}{x}$

මිශ්‍ර ගැටළු

01. $\lim\limits_{x \to 0} \dfrac{1 - \cos(4 \sin x)}{x^2}$

02. $\lim\limits_{x \to 0} \dfrac{1 - \cos^2(2 \sin x)}{1 - \cos 2x}$

03. $\lim\limits_{x \to 0} \dfrac{\tan 5x + \sin 3x - 2 \sin x}{x + \sin 3x}$

04. $\lim\limits_{x \to 0} \dfrac{1 - \cos[1 - \cos 4x]}{x^4}$

05. $\lim\limits_{x \to 0} \dfrac{(1 + x)^7 - 1}{\sin x}$

06. $\lim\limits_{x \to 0} \dfrac{(1 + 4x)^5 - 1}{\sin 3x}$

07. $\lim\limits_{x \to 0} \dfrac{(1 - \cos 2x)(1 - \cos 4x)(1 - \cos 6x) \ldots\ldots (1 - \cos 2x)}{x^{2n}}$

08. $\lim\limits_{x \to a} \dfrac{(x + 2)^{\frac{5}{3}} - (a + 2)^{\frac{5}{3}}}{(x + 3)^{\frac{5}{2}} - (a + 3)^{\frac{5}{2}}}$

09. $\lim\limits_{x \to a} \left[\dfrac{\sqrt{5 + x^2} - \sqrt{5 - x^2}}{\sqrt{x(x + 20)} - 2\sqrt{5x}}\right]$

10. $\lim\limits_{x \to k} \left(\dfrac{\sqrt{4x - k} - \sqrt{2x + k}}{\sqrt{x^2 + k^2} - \sqrt{2k}}\right)$

11. $\lim\limits_{x \to 1} (1 - x) \cot \pi x$

12. $\lim\limits_{x \to 0} \dfrac{1 - \cos\left[\ln|x^3 + 3x^2 + 3x + 1|\right]}{\sin\left[2 \ln|x + 1|^{\frac{3}{2}}\right]}$

13. $\lim\limits_{x \to 2} \dfrac{\log_{10}(x - 1)}{(x - 2)}$

14. $\lim\limits_{x \to 0} \left(\dfrac{x^5 - a^5}{a^3 - x^3}\right) = -\dfrac{1}{15}$ නම් a සොයන්න. $(a > 0)$

15. $\lim\limits_{x \to 0} \dfrac{1}{x} \cos^{-1}\left(\dfrac{1 - x^2}{1 + x^2}\right)$

16. $\lim\limits_{x \to 0} \left(\dfrac{3^x - 1}{\sqrt{2x + 3} - \sqrt{3}}\right)$

17. $\lim\limits_{x \to a} \left[\dfrac{\sin(3x - a) - \sin(x + a)}{x^2 - (a + b)x + ab}\right]$

17. $\lim\limits_{x \to \alpha} \left(\sqrt{a^2x^2 + ax + 1} - \sqrt{a^2x^2 + 1}\right)$

18. $ax^2 + bx + c = 0$ හි මූල α, β නම් $\lim\limits_{x \to \alpha} \left(\dfrac{ax^2 + bx + c}{\sqrt{3x - \alpha} - \sqrt{x + \alpha}}\right)$ අගයන්න.